



**Winter assignment 2022**

**Subject:** Mathematics.

**Class:** 9<sup>th</sup>

**Chapter name:** Number system.

## INTRODUCTION TO NUMBER SYSTEM

The collection of numbers is called the number system. These numbers are of different types such as natural numbers, whole numbers, integers, rational numbers and irrational numbers. Let us see the table below to understand with the examples.

**Natural numbers:** Natural numbers are those numbers which are used for counting. Thus these numbers are also called as counting numbers. Natural numbers is a set or collection of numbers which start from 1. These numbers lie on the right side of the number line and are positive. The group or set of natural numbers is represented by '**N**'.

Therefore,  $N = \{1, 2, 3, 4, 5, 6, \dots\}$ .

- Natural numbers are infinite.
- The least natural number is 1.
- The largest natural number does not exist because every natural number has a successor.

**Whole numbers:** whole numbers are those numbers which start from 0. These numbers lie on the right side of the number line from 0 and are positive. The set of whole numbers is represented by '**W**'.

Therefore,  $W = \{0, 1, 2, 3, 4, 5, 6, \dots\}$ .

Clearly, every natural number is whole number but every whole number is not a natural number. Because 0 is a whole number but not a natural number.

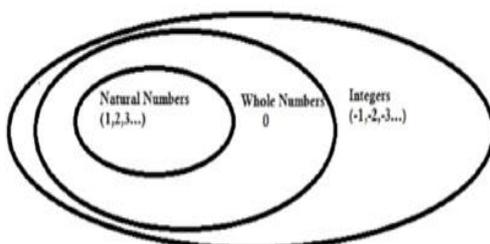
**Integers:** the collection of natural numbers together with 0 and negatives of natural numbers is known as integers. Thus, Integers are the whole numbers which can be positive, negative or zero.

The set or collection of integers is represented by '**I**' OR '**Z**'.

Therefore,  $I = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$ .

- Integers are infinite both the ways.
- The least and the largest integer does not exist because every integer has a predecessor and a successor.
- Integers which are greater than 0 are known as **Positive integers**.
- Integers which are less than 0 are known as **Negative integers**.
- Integers which are greater than or equal to 0 are known as **Non negative integers**.
- Integers which are less than or equal to 0 are known as **Non positive integers**.

Clearly, every natural number and a whole number is an integer.



**Rational numbers:** Rational numbers are those numbers which can be put in the form of  $p/q$ ; where  $p$  and  $q$  are integers and  $q \neq 0$ . Here  $p$  is known as numerator and  $q$  is known as denominator. The collection of rational numbers is represented by 'Q'

Examples of rational numbers are:  $1/2, 3/8, 35/6, 1/1, 2/2, -4/9, -19/-45, 0, 0/67$

$3/\sqrt{2}$  is not a rational numbers because  $\sqrt{2}$  is not an integer.

- Every whole number is a rational number because every whole number can be put in the form of  $p/q$  by dividing them by 1.  
Ex:  $0/1, 1/1, 2/1, 3/1, 4/1, 5/1 \dots$
- Every natural number is a rational number, number because every natural number can be put in the form of  $p/q$  by dividing it by 1.  
Ex:  $1/1, 2/1, 3/1, 4/1, 5/1, \dots$
- Every integer is a rational number, number because every integer can be put in the form of  $p/q$  by dividing it by 1.  
Ex:  $\dots, -4/1, -3/1, -2/1, -1/1, 0/1, 1/1, 2/1, 3/1, 4/1, 5/1, \dots$

Note that there are infinite rational numbers between any two rational numbers.

**Irrational numbers:** all those numbers which are not rational are known as irrational numbers. That is a number is called irrational, if it cannot be written in the form  $p/q$ , where  $p$  and  $q$  are integers and  $q \neq 0$ .

Example:  $\sqrt{2}, \sqrt{3}/2, \sqrt{8}/\sqrt{7}, \sqrt{10}, 2-\sqrt{3}$  etc.

Note that numbers like  $2/0$  are neither rational nor irrational numbers, because division by 0 is not defined.

**Real numbers:** All the numbers which can be represented by a unique point on the number line. A real number is either a rational or irrational. The collection of real numbers is represented by 'R'

**Note that:**

- Every rational number is a real number but every real number is not a rational number.
- Also every irrational number is a real number but every real number is not a an irrational number.
- The square roots of all positive integers may be rational or irrational.  
Ex:  $\sqrt{4} = \pm 2$  which is rational  
But  $\sqrt{7}$  is not an integer or rational. Thus  $\sqrt{7}$  is irrational.  
A number whose square is non-negative is called a real number.
- Real numbers follow Closure property, associative law, commutative law, the existence of an additive identity, existence of additive inverse for Addition.
- Real numbers follow Closure property, associative law, commutative law, the existence of a multiplicative identity, existence of multiplicative inverse, Distributive laws of multiplication over Addition for Multiplication.

Natural Numbers	N	1, 2, 3, 4, 5, ...
Whole Numbers	W	0, 1, 2, 3, 4, 5...
Integers	Z	..., -3, -2, -1, 0, 1, 2, 3, ...
Rational Numbers	Q	$p/q$ form, where $p$ and $q$ are integers and $q$ is not zero.
Irrational Numbers		Which can't be represented as rational numbers

**DECIMAL**

**REPRESENTATION OF RATIONAL AND IRRATIONAL NUMBERS.**

## ❖ Decimal Representation Of Rational

The decimal representation of a rational number is converting a rational number into a decimal number that has the same mathematical value as the rational number. A rational number can be represented as a decimal number with the help of the long division method. We divide the given rational number in the long division form and the quotient which we get is the decimal representation of the rational number. A rational number can have two types of decimal representations (expansions):

- Terminating
- Non-terminating but repeating

Thus, decimal representation of rational numbers is either terminating or non-terminating but repeating.

Let's try to understand what are terminating and non-terminating terms. While dividing a number by the long division method, if we get zero as the remainder, the decimal expansion of such a number is called terminating. And while dividing a number, if the decimal expansion continues and the remainder does not become zero, it is called non-terminating.

**Example 1.** Example:  $1/2$

Let us see the long division of 1 by 2 in the following image:

	0.	5		
2	1.	0		
	0			
	1	0		
	1	0		
		0		

Therefore,  $1/2 = 0.5$  is a terminating decimal

**Example 2:**  $1/3$

$$\begin{array}{r} .333 \text{ recurring} \\ 3 \overline{) 1.000} \\ \underline{-9} \phantom{00} \\ 10 \phantom{0} \\ \underline{-9} \phantom{0} \\ 10 \\ \underline{-9} \\ 1 \text{ non terminating} \end{array}$$

$1/3 = 0.33333\dots$  is a recurring, non-terminating decimal. You can notice that the digits in the quotient keep repeating

**Example 3:** The decimal representation of  $10/2 = 5$  which is terminating.

**Example 4:** The decimal representation of  $14/11 = 1.272727\dots$  which is non-terminating but repeating.

## Some Special Characteristics of Rational Numbers

- Every Rational number is expressible either as a terminating decimal or as a repeating decimal.
- Every terminating decimal is a rational number.
- Every repeating decimal is a rational number

- Terminating decimal expansion means that the decimal representation or expansion terminates after a certain number of digits.
- Every non-terminating but repeating decimal representation corresponds to a rational number even if the repetition starts after a certain number of digits.

### ❖ Decimal Representation Of Irrational Numbers

The decimal representation of irrational numbers is non-terminating and non-repeating

**Example 1:** 0.0100100001001...

**Example 2:** Decimal expansion of  $\sqrt{2} = 1.414213562373095\dots$  which is non-terminating and non-repeating.

### Properties of Irrational Numbers

- These satisfy the commutative, associative and distributive laws for addition and multiplication.
- Sum of two irrationals need not be irrational.

Example:  $(2 + \sqrt{3}) + (4 - \sqrt{3}) = 6$

- Difference of two irrationals need not be irrational.

Example:  $(5 + \sqrt{2}) - (3 + \sqrt{2}) = 2$

- Product of two irrationals need not be irrational.

Example:  $\sqrt{3} \times \sqrt{3} = 3$

- The quotient of two irrationals need not be irrational.

$2\sqrt{3}/\sqrt{3} = 2$

- Sum of rational and irrational is irrational.
- The difference of rational and irrational number is irrational.
- Product of rational and irrational is irrational.
- Quotient of rational and irrational is irrational.

## Difference between Terminating and Recurring Decimals

Terminating Decimals	Repeating Decimals
If the decimal expression of $a/b$ terminates. i.e, comes to an end, then the decimal so obtained is called Terminating decimals.	A decimal in which a digit or a set of digits repeats repeatedly periodically is called a repeating decimal.
Example: $1/4 = 0.25$	Example: $2/3 = 0.666\dots$

### RATIONALISATION

If we have an irrational number, then the process of converting the denominator to a rational number by multiplying the numerator and denominator by a suitable number is called rationalisation.

Example:

$$3/\sqrt{2} = (3/\sqrt{2}) \times (\sqrt{2}/\sqrt{2}) = 3\sqrt{2}/2$$

### Laws of Radicals

Let  $a > 0$  be a real number, and let  $p$  and  $q$  be rational numbers, then we have:

i)  $a^p \cdot a^q = a^{p+q}$

ii)  $(a^p)^q = a^{pq}$

iii)  $a^p/a^q = a^{p-q}$

$$\text{iv) } a^p \times b^p = (ab)^p$$